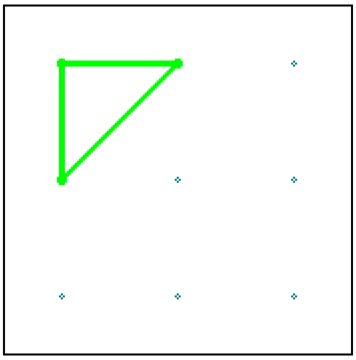
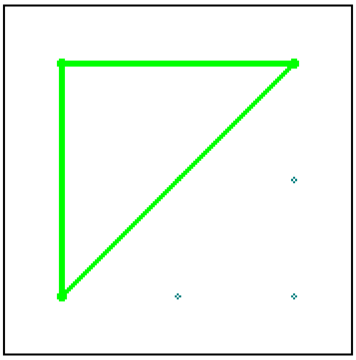
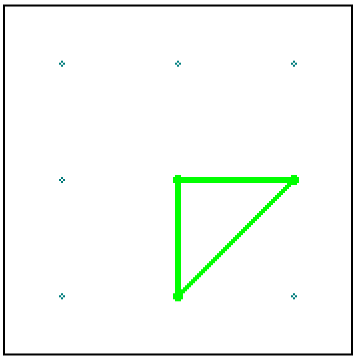
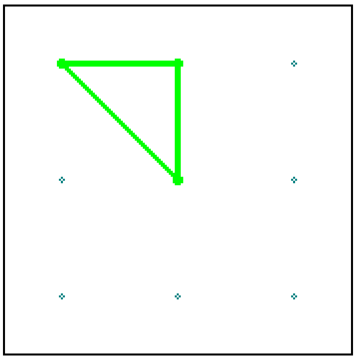
Different triangles

Move the slider to get a *3 by 3 Geoboard*.

Find all the different triangles which can fit on a 3 by 3 geoboard where each corner of the triangle is on a dot. Use dotty paper to draw each of the triangles.

State clearly when you consider triangles to be ‘different’ (stating when two triangles are the ‘same’ can help in clarifying when triangles are different!). For example, are all these ‘different’ or do you consider some or all of these to be the ‘same’?

Can you be sure that you have got <u>all</u> the possibilities? Try to prove that you have them all and that there cannot be any more.

Repeat the above with a 4 by 4 geoboard. 5 by 5?…

Triangle copies

Move the slider to get a *3 by 3 Geoboard*.

Choose a triangle on the 3 by 3 geoboard.

Draw it on dotty paper.

Find all the possible copies of this triangle which can be on the 3 by 3 geoboard with all three corners on a dot. Each copy must be the same size and have the same angles. It can be in a different position or at a different angle.

How many copies of your triangle can you find (counting the original as well)?

Draw all the copies on the same 3 by 3 geoboard.

What patterns can you find in this final drawing?

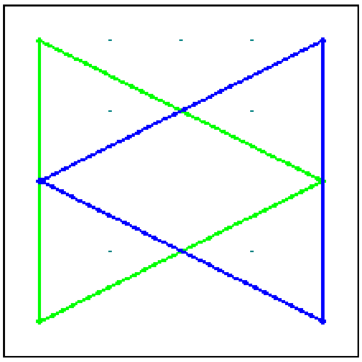
Find a different triangle to start with. Can you find one which has a different number of possible copies?

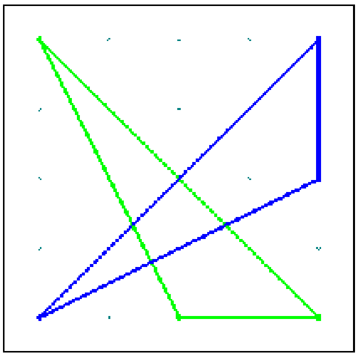
Which triangle(s) has the smallest number of possible copies (counting the original)?

Which triangle(s) has the largest number of possible copies (counting the original)?

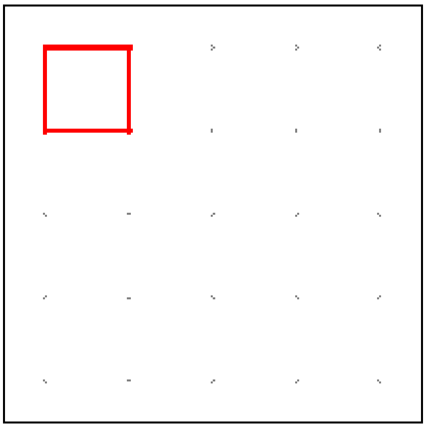
Overlap areas

Move the slider to get a *5 by 5 Geoboard*. On the 5 by 5 geoboard, put two copies of the same triangle so that they overlap. For example:





The idea is to work out the area of the overlap, given that the following square has an area of 1 square unit:



Some overlap areas are reasonably easy to work out and others are very hard! Choose your triangle and how to position the two copies of it so that you can find out the area of the overlap.

On dotted paper, draw where you have put the two copies of the triangle and how you have worked out the area of the overlap.

Repeat the above and try to find as many different overlap areas as you can work out!

Obtuse angled triangles

Move the slider to get a *5 by 5 Geoboard*.

How many different obtuse angled triangles are there on a 5 by 5 geoboard where each triangle has its corners on a dot?

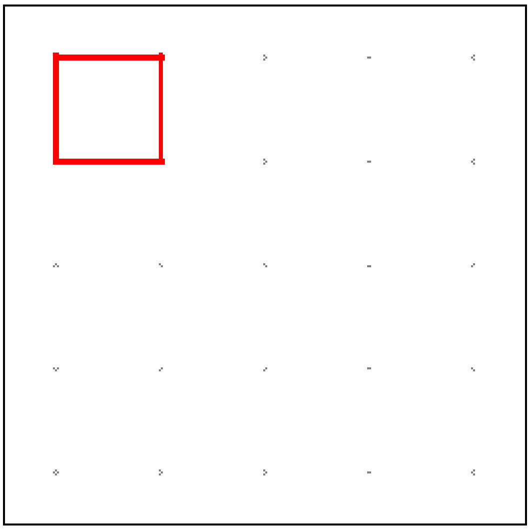
Draw all of them on dotted paper.

Can you be sure that you have got all the possibilities? Try to prove that you have them all and that there cannot be any more.

Areas of triangles

Move the slider to get a *5 by 5 Geoboard*.

If the following square has an area of 1 square unit:



Then try to find a triangle on a 5 by 5 geoboard which has…

… an area of ½;
… an area of 1;
… an area of 1½;
… an area of 2;
… an area of 2½;
… an area of 3;
… etc…
(how far can you go?)

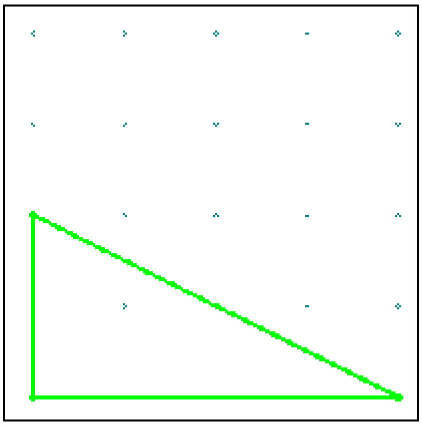
Draw each triangle on dotty paper, indicating its area.

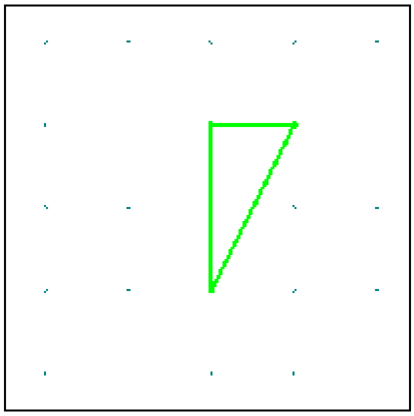
Right-angled triangles

Move the slider to get a *5 by 5 Geoboard*.

How many different right-angled triangles can be made on a 5 by 5 geoboard with each corner on a dot?

Two right-angled triangles will be considered the same if one is exactly the same size as the other but in a different position, or one is an enlarged copy of the other. So the following right-angled triangles would be considered the same as one is exactly twice the size of the other:





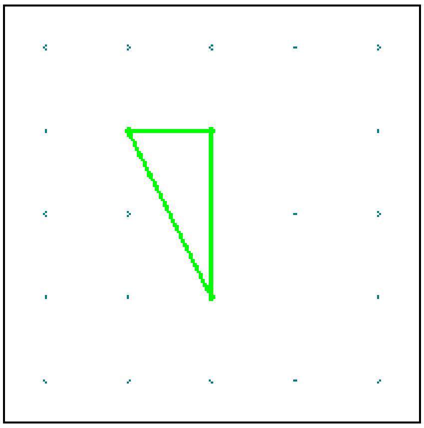
Draw each one on dotty paper.

Do not forget ones which do not have vertical or horizontal sides.

Find the triangle

Move the slider to get a *5 by 5 Geoboard*.

Given that the following triangle has sides of length 1, 2 and $\sqrt{5}$:



Find each of the following triangles on a 5 by 5 geoboard and draw them on dotty paper:

1. an isosceles right-angled triangle with sides of length 2, 2 and $\sqrt{8}$;
2. a right-angled triangle with a perimeter of 12 units;
3. an obtuse-angled triangle with an area of 3 square units;
4. an isosceles right-angled triangle with an area of 4 square units;
5. a right-angled triangle with one side of $\sqrt{10}$ and an area of 2½ square units;
6. an equilateral triangle with an area of 7½ square units;
7. a scalene triangle with two sides of length $\sqrt{13}$ and $\sqrt{10}$, and an area of 5½ square units;
8. a scalene triangle with perimeter of $9 + \sqrt{17}$ units;
9. an obtuse angled triangle whose perimeter is 16.653 units to 3 decimal places;
10. an obtuse-angled triangle whose perimeter is $3(\sqrt{5} + 1)$.

(By the way, one of the above is impossible on a 5 by 5 geoboard! Find out which one!)

Two to one ratio

Move the slider to get a *5 by 5 Geoboard*.

On a 5 by 5 geoboard, find a triangle where one side is twice as long as another side.

How many different triangles can you make like this? Draw a copy on each one on dotty paper.

Can you make one without a right-angle?

If so, draw it.
If not, try to explain why.

Three isosceles triangles

Move the slider to get a *5 by 5 Geoboard*.

Using a 5 by 5 geoboard, find three different isosceles triangles which have the same area. Draw them on dotty paper.

(In this task, ‘different’ will mean the triangles having different angles to each other)

Isosceles triangles

On a 3 by 3 geoboard, find all the different isosceles triangles.

Draw each one on dotty paper.

When you are sure you have them all, explain why you are certain there are no more.

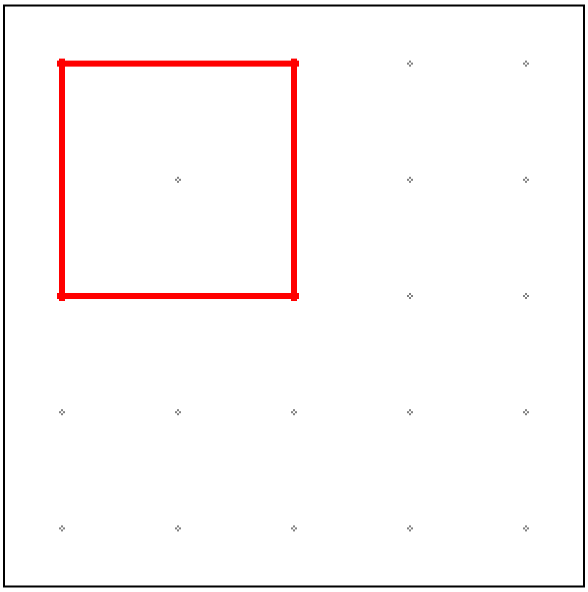
Repeat for a 4 by 4 geoboard.

Lastly repeat for the 5 by 5 geoboard.

Area of four

Move the slider to get a *5 by 5 Geoboard*.

If the following square has an area of 4 square units:



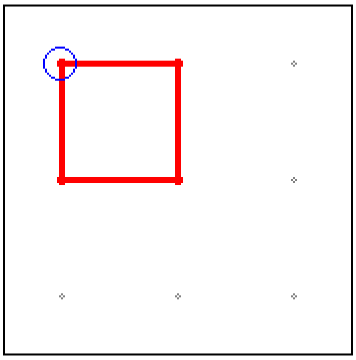
Then find other quadrilaterals on a 5 by 5 geoboard which also have an area of 4 square units.

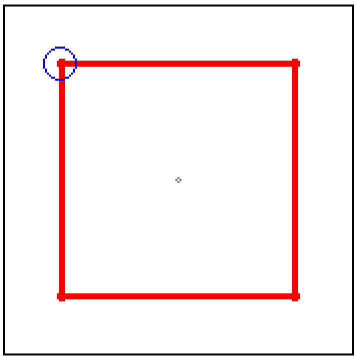
On dotted paper draw as many as you can.

Squares on a point

Move the slider to get a *3 by 3 Geoboard*.

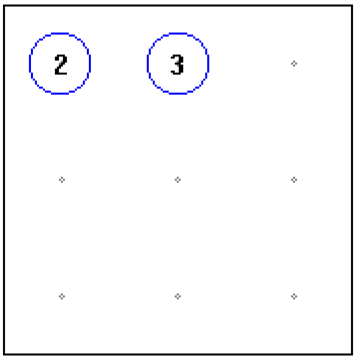
On a 3 by 3 geoboard, there are two squares which have a corner at the top left dot (circled):





There are 3 squares which have a corner at the top middle dot - find them and draw them on dotted paper.

The number of squares which have a corner at these two dots is shown by writing the numbers on top of the dots:



Continue by finding the number of squares which have a corner at each of the remaining dots on the geoboard.

Repeat for the 4 by 4 and then 5 by 5 geoboards.

What connection have these numbers to the total number of squares which can be made on these sized geoboards?

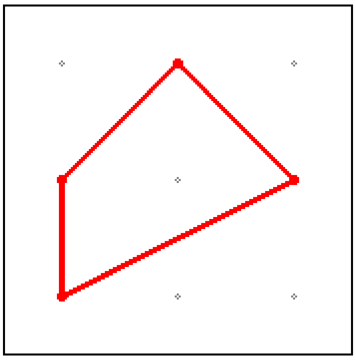
No special name

Move the slider to get a *3 by 3 Geoboard*.

Several quadrilaterals have special names:

* Trapezium
* Parallelogram
* Kite
* Cyclic
* Rhombus
* Rectangle
* Square
* Arrowhead

On a 3 by 3 geoboard, there are only 3 different quadrilaterals which would NOT be called any of the above special names. One of them is:



Copies of this shape which have the same angles are not counted as different. There are two other quadrilaterals. Find them and draw each one on dotty paper.

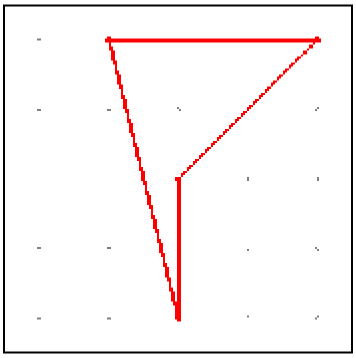
Find how many different quadrilaterals there are on a 4 by 4 geoboard which would NOT be called any of the above special names. Draw each one on dotty paper.

Repeat for the 5 by 5 geoboard.

Find the quadrilateral

Move the slider to get a *5 by 5 Geoboard*.

Given that the following quadrilateral has sides of length 3, $\sqrt{8}$, 2 and $\sqrt{17}$:



Find each of the following quadrilaterals on a 5 by 5 geoboard and draw them on dotty paper:

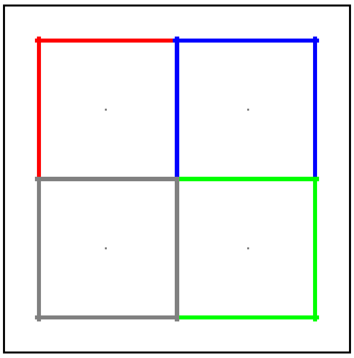
1. a square of area 10 square units;
2. a quadrilateral, which is <u>not</u> a trapezium or a parallelogram, and which has an area of 8 square units;
3. a concave quadrilateral which has a perimeter of $3\sqrt{5}+1$ units;
4. a parallelogram which has a perimeter of $2(\sqrt{5}+3)$ units;
5. a kite, which is <u>not</u> a rhombus, and which has an area of 9 square units;
6. a rhombus which has a perimeter of $4\sqrt{10}$ units;
7. a rectangle which has a perimeter of $6\sqrt{2}$ units;
8. a parallelogram which has an area of 3 square units and one of its sides of length $\sqrt{5}$ units;
9. an arrowhead which has an area of 1 square unit;
10. a cyclic quadrilateral which is <u>not</u> a trapezium or a parallelogram.

(By the way, one of the above is impossible on a 5 by 5 geoboard! Find out which one!)

Covering the 5 by 5 geoboard

Move the slider to get a *5 by 5 Geoboard*.

Obviously, it is possible to have four copies of a square (with an area 4 square units) to completely cover the area of 16 square units inside a 5 by 5 geoboard:



Find four copies of a different quadrilateral which also completely covers the inside of a 5 by 5 geoboard with no gaps and no overlaps. Try to find yet another quadrilateral which does the same.

Arrange two copies each of two different quadrilaterals (making four shapes in all), each with an area of 4 square units, so that they too completely cover the 16 square units inside a 5 by 5 geoboard? Can you find a different solution with two copies of two other quadrilaterals?

Can you arrange four different quadrilaterals, each with an area of 4 square units, so that they completely cover the area of 16 square units inside a 5 by 5 geoboard?

Cover the inside of the 5 by 5 geoboard (no gaps, no overlaps) with four quadrilaterals where:

- the first has an area of 1 square unit;
- the second has an area of 3 square units;
- the third has an area of 5 square units; and
- the fourth has an area of 7 square units?

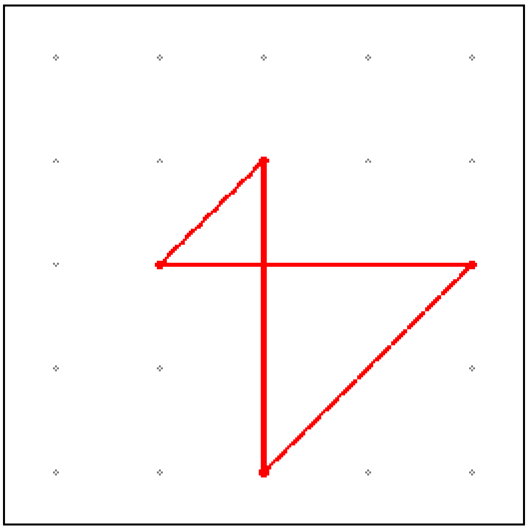
Consider other combinations of four whole numbers which add up to 16, such as: 1, 2, 6 and 7. Which combinations is it possible for four quadrilaterals with those areas to completely cover the 5 by 5 geoboard?

Symmetry

Move the slider to get a *5 by 5 Geoboard*.

On a 5 by 5 geoboard, how many different quadrilaterals can you make which have only one line of symmetry?

Crossed shapes (such as the one below) are not allowed.



Draw each one on dotty paper.

What special name is given to all of these?

Now, draw all the different quadrilaterals which have rotational symmetry of order 2 (and no other order of symmetry, such as order 4).

What special names is given to these?

45 degree angles

Move the slider to get a *4 by 4 Geoboard*.

On a 4 by 4 geoboard, make as many quadrilaterals as possible which have exactly one angle equal to 45 degrees.

Draw each one on dotty paper.

When you are sure you have them all, explain how you know there are no more.

Find all the quadrilaterals which have exactly two 45 degree angles. Again, draw each one on dotty paper and explain how you know there are no more.

